WEILAND (C.)

A CRITICAL STUDY OF THE MAIN DEFECTS OF JAVAL'S OPHTHALMOMETER.

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JAVAL'S ophthalmometer was first brought before the profession about a decade ago with the claim that it enabled the ophthalmologist to determine corneal astigmatism objectively to within a quarter of a diopter. This claim seems now to be conceded by many members of the profession; but I think it will be clear after the following exposition that such a claim is *untenable*, especially according to Javal's own mathematic method, and that it is only by accident (*sit venia verbo!*) that this claim has been partially substantiated in practice.

To understand this seemingly paradoxic statement I must ask my readers to glance at a few mathematic formulæ. First, however, let us recall the method of finding the radius of curvature of a convex mirror, such as the anterior part of the cornea is.

If A B (Fig. 1) be the object, at a right angle to the optic axis E D; if M N be the surface of the convex mirror, the center of which lies at D, and

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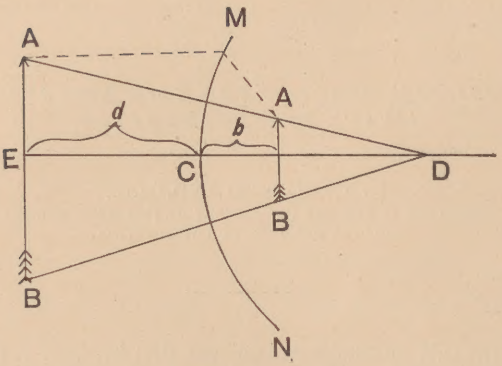
FIG. 1.



if E C $= d$ and the distance of the image A′ B′ from M N $= b$, then we have

$$\text{(I)} \quad \frac{1}{d} - \frac{1}{b} = -\frac{2}{r} \text{ (if } r = \text{C D)},$$

and further,

$$\text{(II)} \quad \frac{\text{A B}}{\text{A}' \text{B}'} = \frac{d+r}{r-b} = \frac{\text{Size of object}}{\text{Size of image}} = \frac{\text{O}}{\text{I}}.$$

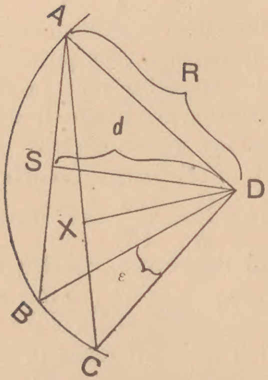
If we express r in equation (II) in terms of d and b of (I), we get $\frac{\text{O}}{\text{I}} = \frac{d}{b}$, or expressing here b in terms of d and r by equation (I) we have $\frac{\text{O}}{\text{I}} = \frac{2d+r}{r}$ (III)

Now, here lies the *first inaccuracy of Javal*, for he makes $\frac{\text{O}}{\text{I}} = \frac{2d}{r}$, which is right only if the object lies so far from the mirror that its image falls at the focus of the mirror, which is at $\frac{r}{2}$. Really, Helm-

holtz did use the formula $\frac{O}{I} = \frac{2\,d}{r}$ because he placed his object at 2000 mm. from the cornea, while Javal makes $d =$ only 280 mm. To prove the inaccuracy we have only to remember that Javal's instrument is so constructed that the image, I, always $= 3$ mm. as soon as the two reflectors have been moved so that their two inner images touch. Therefore, $\frac{O}{I} = \frac{2\,d}{r}$ becomes $\frac{O}{3} = \frac{560}{7.8}$, as r, the average radius of curvature of the cornea, equals 7.8 mm. Then $O = 216$, while by using $\frac{O}{I} = \frac{2\,d + r}{r}$ we get $O =$ 219 mm., which is a difference of about 1.5 per cent.

Another inaccuracy of Javal's instrument arises from the fact that his reflectors slide on an arc, for this sliding changes d continually, as will be easily seen by the following diagram (Fig. 2). Here A B C is the arc on which the reflectors are moved. Suppose, now, that the one slider has to be moved from B to C to get contact of their images, then d will change also. It will become R $\cos\left(\alpha + \frac{\varepsilon}{2}\right)$, if $R = AD$, or the distance of the arc from the cornea, and α the primary angle A D S, and ε the angle between B D and C D. Javal, however, makes $d = R \cos \alpha$ a constant quantity all through his calculation. One may object to this criticism as being very trivial, and, indeed, numerically it does not amount to much; but it must be mentioned here, because it can be so easily remedied that it is astonishing that it escaped Javal's mind.

FIG. 2.



The *main error of Javal*, however, lies in the following: In his own description of his instrument he states that f, the focal distance of the cornea, $= \frac{r}{n - 1}$, if r is the radius of the cornea and n the index of refraction of the cornea and aqueous humor. Now, as we want to find the refractive power of the cornea for rays coming from the outer world, it is undoubtedly true that f can only $= \frac{r\,n}{n - 1}$, for this alone tells us how far back from the anterior surface of the cornea parallel rays from distant objects would meet. Or, if we express it in diopters, then we must say, the refractive power of the cornea $D = \frac{1000}{f} = \frac{1000\,(n - 1)}{r\,n}$. *This* is the right formula, and *not Javal's*, whose expression for

D is $D = \frac{1000\,(n - 1)}{r}$. The following table will illustrate the difference:

43.1 D of Javal ought to be	32.3 D	for $r =$	7.8 mm.
40.5 D " " "	30.3 D	for $r =$	8.3 mm.
37.1 D " " "	27.7 D	for $r =$	8.8 mm.
30 D " " "	22.5 D	for $r =$	11.2 mm., etc.

Now, it may be said that the question is not so much as to the absolute refractive power of the cornea, but the difference between those powers in the different meridians, or the amount of astigmatism. But it must be replied that the amount of astigmatism is affected also, as is shown by the following examples:

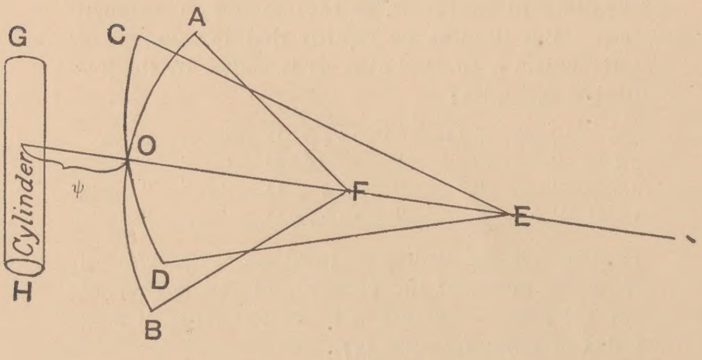
6 D astig. of Javal only	$=$ 4.6 D	astig. of cornea.
3.4 D " " "	$=$ 2.6 D	" "
2.6 D " " "	$=$ 2 D	" "
1 D " " "	$=$ 0.74 D	" "

In short, all the values of Javal for the amount of refractive power of the cornea and the amount of corneal astigmatism are n times too large, if $n =$ index of refraction $= 1.337$.

"But how is that?" the impatient reader will ask. "Does not clinical experience show that Javal's instrument does often give very good results?" I answer, Yes, often, but not always; and must add that this result is not obtained by the right calculation, but results very strangely from accidental causes, as follows: It is well known that the value of a lens changes with regard to its optic effect on the eye according to the position the lens occupies. Now, if Javal were right in his calcula-

tion, so that his instrument showed the real astigmatism of the cornea, then quite a different glass would have to be used at 15 mm. from the cornea, the anterior focal point of the eye, where our lenses ought to be placed. Still we use Javal's numbers even at that distance from the eye, which shows plainly that his calculations for the integral corneal astigmatism must be wrong. To make this more clear let us look at the following figure (Fig. 3): Let A B

FIG. 3.



be the vertical and C D the horizontal meridians of the cornea, which are of differing curvatures, so that F is the focus of the rays through the vertical and E the focus of those through the horizontal meridian. If the refractive power of the cornea in the horizontal meridian = D diopters, and that of the vertical = D′ diopters, if D′ is supposed to be greater than D, what lens have we to place at the anterior focal point of the eye at the distance of ϕ mm. from the cornea, or, more correctly, from

the first principal point of the eye, in order to make the combined refractive power of a cylindrical lens and the cornea in the horizontal meridian equal to that same power in the vertical meridian? To simplify the problem, however, let us put the cylinder, not at ϕ, but at the *anterior* focal point of the *cornea*, which is $= \frac{r}{n - 1} = \frac{7.8}{0.437} = 23$ mm., while ϕ would be about 15 mm. from the cornea. Now, it is well known that if a lens is placed at the anterior focal point of an optic system, the second principal point, and with it the second principal focus, of the combined system is obtained by moving these points of the given system in the same direction through the same definite extent. In our example the second principal point lies, together with the first principal point, at the apex of the cornea, and it is moved through $\frac{\psi \psi'}{f}$ mm. away from the cornea, if ψ and ψ' are the anterior and posterior foci of the cornea and f the focal distance of lens or cylinder. As we want the rays through C D also to come to a focus at F, we have to choose our lens so that

$$F E = \frac{\psi \psi'}{f} = \frac{\psi \times 1000}{f D}, \text{ because } D = \frac{1000}{\psi'}.$$

This we may write

$F E = \frac{\psi D_L}{D}$, if we mean by $D_L = \frac{1000}{f}$, the refractive power of the cylinder in that meridian. Further, it is clear that F E, the focal interval, $= \frac{1000}{D} - \frac{1000}{D'}$,

as $OE = \frac{1000}{D}$ and $OF = \frac{1000}{D'}$. Therefore, we have now $\frac{1000}{D} - \frac{1000}{D'} = \frac{\psi D_L}{D} = \frac{1000 (D' - D)}{D D'}$

And as $\psi = \frac{r}{n - 1}$ and $D = \frac{1000 (n - 1)}{r n}$, we can write for $\frac{\psi}{D}$ the value $\frac{1000}{D^2 n}$, so that we have

$$\frac{1000 (D' - D)}{D D'} = \frac{1000 D_L}{n D^2}, \text{ or } \frac{D' - D}{D'} = \frac{D_L}{n D}.$$

Therefore, $D_L = \frac{n D (D' - D)}{D'}$, if $D' - D =$ the amount of real corneal astigmatism $= d$. The value of our lens, therefore, which, at the anterior focal point of the cornea $= 23$ mm., corrects the corneal astigmatism, is $D_L = \frac{n D}{D'} d = \frac{n (D' - d)}{D'} d$. This we may write $D_L = \frac{n D}{n D'} n d$; and as we know from former considerations that the real values of astigmatism and corneal diopters have to be multiplied by n to get those of Javal, and if the respective values of Javal are called Δ, Δ', and δ, we have $D_L = \frac{\Delta}{\Delta'} \delta = \frac{\Delta' - \delta}{\Delta'} \delta = \delta \frac{\Delta}{\Delta + \delta}$. Now, usually $\frac{\Delta}{\Delta + \delta}$ is not much different from 1, and so we get approximately $D_L = \delta$, if $\delta =$ Javal's astigmatism. Suppose, for example, that there was a real corneal astigmatism of 3 D, then Javal's instrument would indicate an astigmatism of 4Δ, while the real correction at 23 mm. would be a cylinder of 3.6 D. So 1 real D would be 1.337 of Javal $= 1.19$ at 23

mm. if the meridian of lowest curvature was 37.1 Δ. These values would change a little if the lens were placed at 15 mm. away from the cornea—*e. g.*, 3.6 D at 23 mm. would be 3.7 D at 15 mm. In the higher forms of astigmatism, however, the difference is very considerable; for, suppose that Javal's instrument showed 8 Δ astigmatism, then this would equal 6.5 D at 23 mm. from the eye and 6.8 D at 15 mm. away from the eye, so that between Javal's glass and the true cylinder there would be a difference of about 1.25 D, certainly a very considerable amount. In the same way, Javal's 6 would be equal to a 5 cylinder.

Another inaccuracy must be mentioned still, with reference to the graded reflector, where each step is supposed to indicate one diopter. To get the size of those steps Javal uses the formula $\Delta = \frac{1000\,(n - 1)\,O}{2\,d\,I}$ where d is the distance of the object O from the cornea, while the right formula would be

$$D = \frac{1000\,(n - 1)\,(O - I)}{2\,d\,n\,I}.$$

Now, Javal makes *d about* 281 mm. in his latest model (it changes continually, as already proved), and $I = 3$ mm.; and as $n = 1.337$ we get

$$\Delta = \frac{337 \times O}{281 \times 6} \text{ and } 1 = \frac{337 \times O}{281 \times 6} = \frac{O}{5},$$

from which it follows that $O = 5$ mm. for each diopter, while from the true formula

$$D = \frac{1000\,(n - 1)\,(O - I)}{2\,d\,n\,I}$$

it follows that the object O ought to increase 6.7 mm. for each diopter. But while Javal is right in making

the steps of his reflector = 5 mm. each for *his own* diopters, he is wrong in making those reflectors curved, because each step has then less value than 5 mm. as measured in the line of union of the two reflectors.

So far, all the objections raised have nothing to do with the principle of the instrument, but only with the special form given to it by Javal. For it must not be forgotten that the instrument is almost the same as that described by Rochon in the *Journal de Phys.*, vol. liii, in 1801, and that it has a good place in science under the name of Rochon's micrometer. Javal only gave the bi-refracting prism a definite place in the telescope and then made the calculation for the refractive power of the cornea, adding an arc and two reflectors. Now, the calculation and the arc, as well as the two reflectors, are not quite correct, but the error might be easily rectified, and so a more perfect ophthalmometer on Rochon's principle be constructed. But it seems to me that there is a serious defect in the very principle of the instrument, because the bi-refracting quartz-prism is not achromatic, so that the margins of the images are not sharply defined, which makes it impossible to get the accurate contact so necessary for an accurate result.

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